Structure of Matter II
20 June May 2022, 16.00-18.00, Exam Hall 1 A1–H12
Write your name and student number on every sheet
Extra-time students: 10 minutes per hour => 20 min extra

PROBLEM 1. Crystal lattices [12 pts]

Consider a rectangular 3D lattice with the atomic lattice distances in x, y, and z direction equal to a, $2a$ and $3a$, respectively.

a) Calculate the reciprocal unit vectors corresponding to this lattice. [4 pts]

b) Calculate the volumes of the Wigner-Seitz unit cell and the (first) Brillouin zone cell. [3 pts]

c) Consider the planes described by the Miller indices (1,2,3). Determine the distance between these planes. [3 pts]

d) What is Bloch's theorem? [2 pts]

PROBLEM 2. Electrons and crystals [11 pts]

a) Explain why we get band gaps in the energy-momentum relationship of crystals. [4 pts]

b) Do these band gaps become more or less wide when the interaction between the electrons and the atoms increases? explain your answer. [2 pts]

In the course, we have seen that the density-of-states, g, in a 3D electron gas is related to the energy ε by $g(\varepsilon) \propto \sqrt{\varepsilon}$.

c) Now consider a **1D** crystal and derive the allowed k values. [2 pts]

d) Calculate the density-of-states g of such a **1D** electron gas. [3 pts]

Please turn over for Q3

PROBLEM 3. Doping and pn-junctions [13 pts]

Consider a pn-junction in equilibrium.

a) What is the electrical charge (positive, negative, or neutral) of a small volume of the semiconductor in the p-region, far away from the interface? explain your answer. [2 pts]

b) What is the electrical charge (positive, negative, or neutral) of a small volume of the semiconductor in the n-region, within the depletion region (i.e. close to the interface)? explain your answer. [2 pts]

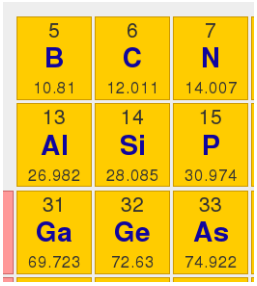
c) Show that the conductivity of a p-doped semiconductor can be written as

$$\sigma = \frac{pe^2\tau}{m^*},$$

where τ is the average time between scattering events and m^* is the effective mass. [3 pts]

d) Suppose you need a piece of p-doped silicon with a conductivity of 100 S/cm. Calculate the required distance between the Fermi level and the valence band edge. Assume τ =1 ns and m^* = 0.1 m_e. [4 pts]

e) This image shows a cut-out of the periodic table of the elements:



Which elements can be used to p-dope silicon? Explain your answer. [2 pts]

Please check whether your name and student number are on every sheet
Please hand in the exam questions (cannot be taken home!)

Some constants

m_e = 9.1096 × 10^{-31} kg
e = 1.6022 × 10^{-19} C
for silicon: N_C =3.22×10^{19} cm^{-3} and N_V=1.83×10^{19} cm^{-3}.
kT = 25 meV (near room-temperature)